Generalized Chaplygin gas as a scheme for Unification of Dark Energy and Dark Matter

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Abstract. We study the cosmological scenario arising from the dynamics of a generalized Chaplygin gas. The equation of state of the system is given in terms of the energy density, ρ, and pressure, p, by the relationship $p = -A/\rho^\alpha$, where A is a positive constant and $0 < \alpha \leq 1$. The conditions under which homogeneity arises are discussed and it is shown that this equation of state describes a universe evolving from a phase dominated by non-relativistic matter to a phase dominated by a cosmological constant via an intermediate period where the effective equation of state is $p = \alpha\rho$.

1 Introduction

Recently, it has been suggested that the change of behaviour of the missing energy density might be controlled by the change in the equation of state of the background fluid instead of the form of the potential, avoiding in this way fine-tuning problems [1]. In the framework of Friedmann-Robertson-Walker cosmology, this is achieved considering an exotic background fluid, the Chaplygin gas, described by the equation of state

$$p = -\frac{A}{\rho^\alpha} \quad , \tag{1}$$

with $\alpha = 1$ and A a positive constant. Introducing this equation of state into the equation resulting from the covariant conservation of the energy-momentum tensor for an homogeneous and isotropic spacetime, leads to a density evolving as

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \quad , \tag{2}$$

where a is the scale factor of the Universe and B an integration constant. It is remarkable that this simple model smoothly interpolates between a dust dominated phase where $\rho \simeq \sqrt{B}a^{-3}$ and a De Sitter phase where $p \simeq -\rho$, through an intermediate regime described, for $\alpha = 1$, by the equation of state for stiff matter, $p = \rho$ [1]. It is interesting that the Chaplygin gas admits a brane interpretation as Eq. (1), with $\alpha = 1$, is the equation of state associated with the parametrization invariant Nambu-Goto d-brane action in a $(d+1,1)$ spacetime. This action leads, in the light-cone parametrization, to the Galileo-invariant Chaplygin gas in a $(d,1)$ spacetime and to the Poincaré-invariant Born-Infeld action in a $(d,1)$ spacetime (see [2] and references therein).

In what follows we discuss the results of our research on the generalized Chaplygin gas with $0 < \alpha \leq 1$ [3].

2 The Model

We consider, as first discussed in Ref. [4], the Lagrangian density for a massive complex scalar field, Φ,

$$\mathcal{L} = g^{\mu\nu}\Phi^*_{,\mu}\Phi_{,\nu} - V(|\Phi|^2) \quad . \tag{3}$$

This scalar field, with mass m, can be expressed as $\Phi = (m\phi/\sqrt{2})\exp(-im\theta)$.

The scale of the inhomogeneity is set assuming that spacetime variations of ϕ correspond to scales greater than m^{-1}, thus

$$\phi_{,\mu} << m\phi \quad . \tag{4}$$

In this (Thomas-Fermi) approximation, the Lagrangian density can be written as

$$\mathcal{L}_{TF} = \frac{\phi^2}{2} g^{\mu\nu}\theta_{,\mu}\theta_{,\nu} - V(\phi^2/2) \quad . \tag{5}$$

Notice, that the field θ can be regarded as a velocity field provided $V' > 0$, i.e.

$$U^\mu = \frac{g^{\mu\nu}\theta_{,\nu}}{\sqrt{V'}} \quad , \tag{6}$$

so that, on the mass shell, $U^\mu U_\mu = 1$. Hence, the energy-momentum tensor built from the Lagrangian density Eq. (5) takes the form of a perfect fluid whose thermodynamic variables can be written as

$$\rho = \frac{\phi^2}{2} V' + V \quad , \tag{7}$$

$$p = \frac{\phi^2}{2} V' - V \quad . \tag{8}$$

Using Eqs. (7) and (8), together with Eq. (1), one finds a relationship between ϕ^2 and ρ:

$$\phi^2(\rho) = \rho^\alpha(\rho^{1+\alpha} - A)^{\frac{1-\alpha}{1+\alpha}} \quad . \tag{9}$$

Further manipulation, substituting Eqs. (7), (8) and (9) into the Lagrangian density (5), shows that it is possible to establish a brane connection

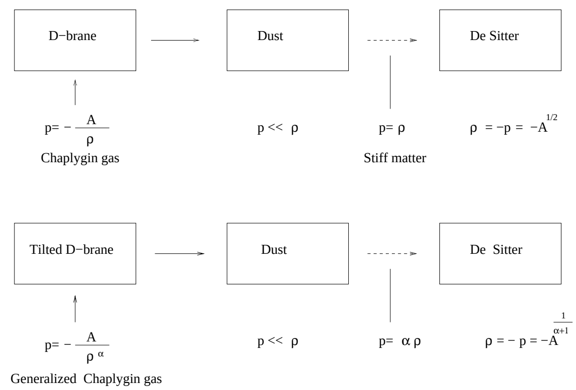

Figure 1: Cosmological evolution of a universe described by a generalized Chaplygin gas equation of state

to this model, as the resulting Lagrangian density has the form of a *generalized* Born-Infeld theory:

$$\mathcal{L}_{GBI} = -A^{\frac{1}{1+\alpha}} \left[1 - (g^{\mu\nu}\theta_{,\mu}\theta_{,\nu})^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \quad . \tag{10}$$

The potential arising from this model can be written as

$$V = \frac{\rho^{1+\alpha} + A}{2\rho^{\alpha}} = \frac{1}{2}\left(\Psi^{2/\alpha} + \frac{A}{\Psi^2}\right) \quad , \tag{11}$$

where $\Psi \equiv B^{-(1-\alpha/1+\alpha)} a^{3(1-\alpha)} \phi^2$, which reduces to the duality invariant, $\phi^2 \to A/\phi^2$, and scale-factor independent potential for the Chaplygin gas.

The effective equation of state in the intermediate phase between the dust dominated phase and the De Sitter phase is obtained expanding Eq. (2) in subleading order:

$$\rho \simeq A^{\frac{1}{1+\alpha}} + \left(\frac{1}{1+\alpha}\right) \frac{B}{A^{\frac{\alpha}{1+\alpha}}} a^{-3(1+\alpha)} \quad , \tag{12}$$

$$p \simeq -A^{\frac{1}{1+\alpha}} + \left(\frac{\alpha}{1+\alpha}\right) \frac{B}{A^{\frac{\alpha}{1+\alpha}}} a^{-3(1+\alpha)} \quad , \tag{13}$$

which corresponds to a mixture of vacuum energy density $A^{\frac{1}{1+\alpha}}$ and matter described by the “soft” equation of state:

$$p = \alpha\rho \quad . \tag{14}$$

In broad terms, the comparison between the cosmological setting we propose and the one emerging from the Chaplygin gas, discussed in Refs. [1, 4], is exhibited in Figure 1.

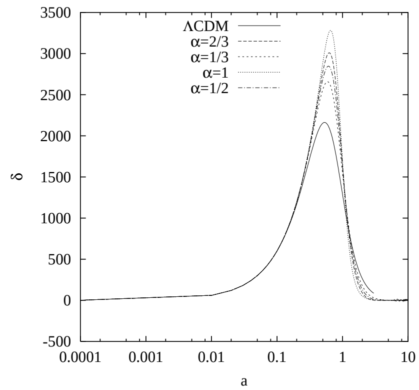

Figure 2: Density contrast for different values of α, as compared with ΛCDM as a function of a relative to the present ($a_0 = 1$)

Next, we analyse the issue of the growth of inhomogeneities in our model. Using the Zeldovich method for the treatment of inhomogeneities, one can write,

$$\rho \simeq \bar{\rho}(1+\delta) \quad , \quad p \simeq -\frac{A}{\bar{\rho}^{\alpha}}(1-\alpha\delta) \quad , \tag{15}$$

where $\bar{\rho}$ is given by Eq. (2), δ is the density contrast. Using the perturbed Friedmann equations, and the unperturbed Raychaudhuri equations, we have solved δ [3] as a function of a for different values of α which we have plotted in Figure 2. Hence, we verify for any α the claim of Refs. [4, 5], for $\alpha = 1$, that the density contrast decays for large a. Figure 2 also shows the main difference in behaviour of the density contrast between a universe filled with matter with a "soft" or "stiff" equations of state as the former resembles more closely the ΛCDM. Research on further observational implications of our results is under way and will be presented elsewhere.

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